A MATHEMATICAL BASIS FOR THE RANDOM DECREMENT VIBRATION SIGNATURE ANALYSIS TECHNIQUE

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ABSTRACT

The mathematical basis for the Random Decrement technique of vibration signature analysis is established. The general relationship between the autocorrelation function of a random process and the Randomdec signature is derived. For the particular case of a linear time invariant system excited by a zero-mean, stationary, Gaussian random process, a Randomdec signature of the output is shown to be proportional to the autocorrelation of the output.

Example Randomdec signatures are computed from acceleration response time histories from an offshore platform.

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INTRODUCTION

The Random Decrement Technique of vibration signature analysis was empirically developed in the late 1960's by Henry Cole [1,2,3,4].

Since that time it has achieved rather widespread use, especially in the aerospace industry, for the analysis of experimentally generated vibration data [3,4,5,6,7]. The method is frequently used for the determination of modal damping ratios and the detection of mechanical failure. The method also forms the basis of another more general vibration analysis technique known as "Ibrahim Time Domain Modal Vibration Testing Technique" [8].

The Randomdec method has achieved rather widespread use because the instrumentation is rather simple, the data processing can be done generally real time, and most of all, because it appears to work. The missing element in the literature and in the day to day interpretation of the results of the technique is a sound mathematical formulation which shows exactly what a randomdec signature is. Without such a theoretical basis the accuracy of, for example, estimates of modal damping ratios of a multiple degree of freedom system cannot be determined.

This paper presents the mathematical basis for the Randomdec technique. The general relationship between the autocorrelation function of a random process and the randomdec signature is derived. A completely general result is obtained. In the particular case of a linear, time invariant system excited by a zero mean, stationary, Gaussian random process a Randomdec signature is shown to reduce to

$$D_{X_o}(\tau) = \frac{R_X(\tau)}{R_X(0)} X_o \tag{1}$$

where $X(t)$ is the response of the linear system

X_o is the trigger level for the acquisition of sample time histories $X(t)$.

$R_X(\tau)$ is the autocorrelation function of the random process $X(t)$

$R_X(0) = R_X(\tau=0)$

This result reveals a small but important difference from the commonly believed interpretation of the Randomdec signature. It is usually accepted that if a linear system is excited by a stationary, Gaussian random process then the Randomdec signature is the same as the free vibration response of that linear system to the set of specified initial conditions. The result of Equation (1) shows that for the case of a specified initial amplitude,

the signature is proportional to the autocorrelation function. The autocorrelation function is not in general proportional to the free vibration decay of the linear system. It happens that for the case of a single degree of freedom oscillator excited by a white noise, stationary, Gaussian process, the autocorrelation function is exactly proportional to the free vibration decay from a specified initial displacement. This special case is often used to argue on intuitive grounds that the Randomdec signatures resulting from non-white Gaussian stationary excitations also represent free decay curves. The error in previous analyses which leads to this conclusion is identified. Fortunately, most applications of the Randomdec method have been circumstances in which the excitation was sufficiently broadband that the free vibration decay interpretation did not lead to substantial errors.

The Definition and Intuitive Theory of the Randomdec Method

A Randomdec signature is simply the trace formed by a waveform averaging a number of specially selected segments from an observed time history. Each of the segments share the common attribute of known initial conditions. Since one may specify an initial value and/or an initial slope for the selected segments, an infinite variety of possibilities exists for the resultant signature. The most popular choice is to only specify the initial amplitude for the segments. For a mechanical system the physical interpretation of such initial conditions is a specification of the initial displacement but not the velocity of the system at the time each segment is selected.

An equivalent definition of the Randomdec signature can be obtained using the concept of ensemble averages. In order to do this one must first assume that the random process is ergodic. Accordingly averages computed from a single time history are equivalent to averages computed across the ensemble of all potential time histories of the process. Under this assumption, the definition of the Randomdec signature is simply the conditional expected value of the random process. In conditioning the expected value, members of the ensemble are excluded from the computation unless they possess the specified values for the initial conditions. These concepts form the basis for the analytical treatment of the Randomdec signature throughout the remainder of the paper.

The intuitive theory of Randomdec is most easily demonstrated by the example of a single degree of freedom mechanical oscillator excited by a zero-mean, stationary, Gaussian random force. The equation of motion of this system is given by

$$M\ddot{X} + R\dot{X} + KX = F(t) \tag{2}$$

The response X(t) is also a zero-mean, stationary, Gaussian process.

Consider a randomly selected segment of the response X(t). At the beginning of the segment, for which t is arbitrarily set to zero, the segment has particular initial values for the amplitude and slope; X(0) = a and $\dot{X}(0) = b$. If at time t=0 the excitation had been removed, then the response X(t) would have been simply the transient decay from the initial conditions a and b.

If on the other hand the excitation had continued, the resulting response would have been the linear superposition of the transient decay due to initial conditions plus the convolution integral of the impulse response function for the system and the excitation as shown.

$$X(t) = a\, e^{-\xi\omega_0 t}\left[\cos \omega_1 t + \frac{\omega_0}{\omega_1} \sin \omega_1 t\right] + \frac{b}{\omega_1} e^{-\xi\omega_0 t} \sin \omega_1 t \; +$$

$$+ \int_0^t h(t - \tau)\, F(\tau)\, d\tau \tag{3}$$

where $\omega_o = \sqrt{K/M}$, $\omega_1 = \omega_o\sqrt{1-\xi^2}$.

As previously stated, the Randomdec signature is simply the average of a large number of segments of the response $X_i(t)$ given that each must start with the same initial conditions, a and b. Using an ensemble average, the Randomdec signature is the conditioned expected value of Equation (3).

$$E[X(t)|a,b] = a\, e^{-\xi\omega_o t}[\cos \omega_1 t + \frac{\omega_o\xi}{\omega_1} \sin \omega_1 t] + \frac{b}{\omega_1} e^{-\xi\omega_o t} \sin \omega_1 t +$$

$$+ \int_0^t h(t-\tau)E[F(\tau)|a,b]\, d\tau \qquad (4)$$

where the expression E[|a,b] should be interpreted as the expected value of the specified random variable, given that a and b are the initial conditions on the response $X_i(t)$.

The intuitive theory of Randomdec and also the solution proposed by Caughey [9] in a paper on earthquake response published in 1961 argue that because the input was specified as a zero mean, stationary, random process then the expected value of the forcing function in the convolution integral of Equation (4) must be zero, thereby proving that the expected value of X(t) given the initial conditions a and b is simply the transient decay of the system from those initial conditions. This is not true. The requirement of known values of the output at t = 0 has biased the expected value of the excitation in such a way that it is no longer necessarily zero, just because it is in general a zero mean input. This is demonstrated below for the general case of a linear, time invariant system excited by a random process, F(t).

The cross correlation of the input at t_2 with an output at t_1 is given by:

$$R_{XF}(t_1,t_2) \;=\; \int_{X_1}\int_{F_2} X_1F_2\; P(X_1,F_2)\; dX_1dX_2 \tag{5}$$

where X_1 and F_2 denote the processes of $X(t_1)$ and $F(t_2)$ and $P(X_1,F_2)$ is the joint probability density function of X_1 and F_2.

The joint pdf can be written in terms of a conditional pdf and a first order pdf as follows:

$$P(X_1,F_2) \;=\; P(X_1)\; P(F_2|X_1) \tag{6}$$

and Equation (5) may be rewritten as

$$R_{XF}(t_1,t_2) \;=\; \int_{X_1} X_1P(X_1) \int_{F_2} F_2\; P(F_2|X_1)\; dF_2dX_1 \tag{7}$$

This integral over F_2 is the expected value of $F(t_2)$ given $X(t_1)$, where, to simplify the example, it was assumed that $\dfrac{dX(t_1)}{dt}$ was not specified. In most cases the cross-correlation is not zero, which can only mean that the expected value of $F(t_2)$ given $X(t_1)$ cannot be, in general, zero.

The intention of this analysis was to establish that the intuitive arguments behind Randomdec and for that matter the earlier analysis of reference [9] are not generally correct. The derivation of the Randomdec signature is presented in the next section.

The General Relationship Between the Autocorrelation Function and the Randomdec Signature

The Randomdec signature is computed by averaging an ensemble of time histories of a random process. The only common feature of the histories is that in each case, the sample has started with the same initial conditions. To simplify the analysis, consider the case that only an initial amplitude but not slope is specified. In probabilistic terms the task is to find the expected value of a random process $X(t)$, evaluated at $t = t_2$, given that at a previous time t_1, the random process had crossed the trigger level, X_o. A mathematical expression of this definition of the Randomdec signature is:

$$D_{X_o}(t_1,t_2) \equiv E[X(t_2)|X(t_1) = X_o] \tag{8}$$

where the expression on the right is the expected value of $X(t_2)$ given $X(t_1) = X_o$, and the expression on the left is by definition the Randomdec signature.

The derivation to follow relates the Randomdec signature to the autocorrelation function of a random process. Since the derivation uses only the definitions of the autocorrelation function and the Randomdec signature the result is entirely general.

The autocorrelation function of a random process $X(t)$ may be defined as follows as shown in the text <u>Random Vibration</u> by Crandall and Mark [10].

$$R_X(t_1,t_2) = E[X(t_1)\ X(t_2)] = \int_{X_1}\int_{X_2} X_1X_2\ p(X_1,X_2)\ dX_1\ dX_2 \tag{9}$$

where the abbreviations X_1 and X_2 denote the random variables $X(t_1)$ and $X(t_2)$. $p(X_1,X_2)$ is the joint probability density function describing the distribution of X_1 and X_2. This joint probability density function may be expressed as the product of a conditional probability density function and a first order probability density function as shown below.

$$p(X_1,X_2) = p(X_2|X_1)\ p(X_1) \tag{10}$$

Substitution into Equation (9) leads to

$$R_X(t_1,t_2) = \int_{X_1} \int_{X_2} X_1\ p(X_1)X_2\ p(X_2|X_1)\ dX_1 dX_2 \tag{11}$$

These two integrals may be computed sequentially as follows:

$$R_X(t_1,t_2) = \int_{X_1} X_1\ p(X_1) \int_{X_2} X_2\ p(X_2|X_1)\ dX_2 dX_1 \tag{12}$$

If X_1 is defined to be the trigger level X_0 then the integral over X_2 yields exactly the expected value of X_2 given $X(t_1) = X_1$ which is the definition of the Randomdec signature as given in Equation (12). Therefore,

$$R_X(t_1,t_2) = \int_{X_1} X_1\ p(X_1)\ E[X_2|X(t_1) = X_1]dX_1 \tag{13}$$

$$R_X(t_1,t_2) = \int_{X_1} X_1\ p(X_1)\ D_{X_1}(t_1,t_2)\ dX_1 \tag{14}$$

An interpretation of Equation (13) is that the autocorrelation function of the random process X(t), computed between any two instants in time t_1 and t_2, is a weighted sum of all possible Randomdec signatures of X(t). The weighting factor is the product of the trigger level X_1 and its probability of occurrance $p(X_1)$ at time t_1.

Results for Stationary Gaussian Random Processes

A specific case for which the mathematics are tractable is a linear, time-invariant system excited by a zero-mean, stationary, but not necessarily white, Gaussian random process. For this case, the system response will also be a zero-mean, stationary, Gaussian random process. The autocorrelation function contains a complete characterization of such a process. The following equations relating the probability distribution to the autocorrelation function can be found in Crandall and Mark.

$$p(X_1) = \frac{1}{\sqrt{2\pi\sigma_1^2}} \exp\left[-\frac{(X_1 - m_1)^2}{2\sigma_1^2}\right] \tag{15}$$

$$p(X_1, X_2) =$$

$$\frac{1}{2\pi\sigma_1\sigma_2\sqrt{1 - \rho_{12}^2}} \exp\left\{\frac{-1}{2(1-\rho_{12}^2)}\left[\frac{(X_1-m_1)^2}{\sigma_1^2} - \frac{2\rho_{12}(X_1-m_1)(X_2-m_2)}{\sigma_1\sigma_2} + \frac{(X_2-m_2)^2}{\sigma_2^2}\right]\right\} \tag{16}$$

For stationary random processes the autocorrelation function depends only on the time difference between t_1 and t_2 and not on t_1 and t_2 individually. Defining this time difference as

$$\tau = t_2 - t_1, \tag{17}$$

setting $X(\tau = 0) = X_o$,

and noting that for the above equations

$$m_1 = m_2 = 0 \tag{18}$$

$$\sigma_1^2 = \sigma_2^2 = R_X(0) \tag{19}$$

$$\rho_{12}(\tau) = \frac{R_X(\tau)}{R_X(0)} \tag{20}$$

these expressions follow:

$$p(X_o) = \frac{1}{\sqrt{2\pi R_X(0)}} \exp\left[\frac{-X_o^{\ 2}}{2R_X(0)}\right] \tag{21}$$

$$p(X_o, X_\tau) =$$

$$\frac{1}{2\pi R_X(0)\sqrt{1-\frac{R_X^{\ 2}(\tau)}{R_X^{\ 2}(0)}}} \exp\left\{\frac{-1}{2\left(1-\frac{R_X^{\ 2}(\tau)}{R_X^{\ 2}(0)}\right)}\left[\frac{X_o^{\ 2}}{R_X(0)} - \frac{2R_X(\tau)X_oX_\tau}{R_X^{\ 2}(0)} + \frac{X_\tau^{\ 2}}{R_X(0)}\right]\right\} \tag{22}$$

The conditional probability density function for $X(\tau)$ given X_o also follows.

$$p(X_\tau | X_o) = p(X_o, X_\tau) \,/\, p(X_o) \tag{23}$$

$$p(X_\tau | X_o) = \frac{1}{\sqrt{2\pi R_X(0)\left(1-\frac{R_X^{\ 2}(\tau)}{R_X^{\ 2}(0)}\right)}} \exp\left\{\frac{-1}{2\left(1-\frac{R_X^{\ 2}(\tau)}{R_X^{\ 2}(0)}\right)}\right.$$

$$\left.\left[\frac{X_o^{\ 2}}{R_X(0)} - \frac{2R_X(\tau)X_oX_\tau}{R_X^{\ 2}(0)} + \frac{X_\tau^{\ 2}}{R_X(0)} - \frac{X_o^{\ 2}}{R_X(0)}\left(1-\frac{R_X^{\ 2}(\tau)}{R_X^{\ 2}(0)}\right)\right]\right\} \tag{24}$$

$$p(X_\tau | X_o) = \frac{1}{\sqrt{2\pi\sigma_a^{\ 2}}} \exp\left\{\frac{-1}{2\sigma_a^{\ 2}}\left[X_\tau^{\ 2} - \frac{2R_X(\tau)}{R_X(0)} X_\tau X_o + \frac{R_X^{\ 2}(\tau)}{R_X^{\ 2}(0)} X_o^{\ 2}\right]\right\} \tag{25}$$

where: $\sigma_a^2 = R_X(0)\left(1 - \frac{R_X^2(\tau)}{R_X^2(0)}\right)$

$$p(X_\tau|X_o) = \frac{1}{\sqrt{2\pi\sigma_a^2}} \exp\left\{\frac{-1}{2\sigma_a^2}\left[X_\tau - \frac{R_X(\tau)}{R_X(0)} X_o\right]^2\right\} \tag{26}$$

From the definition of the Randomdec signature:

$$D_{X_o}(\tau) = \int_{X_\tau} X_\tau p(X_\tau|X_0)\, dX_\tau \tag{27}$$

Therefore:

$$D_{X_o}(\tau) = \int_{X_\tau} X_\tau \frac{1}{\sqrt{2\pi\sigma^2 a}} \exp\left\{\frac{-1}{2\sigma_a^2}\left[X_\tau - \frac{R_X(\tau)}{R_X(0)} X_o\right]^2\right\} dX_\tau \tag{28}$$

But this integral is simply the expected value of a simple Gaussian probability distribution. The result may be obtained by inspection.

$$D_{X_o}(\tau) = \frac{R_X(\tau)}{R_X(0)} X_o \tag{29}$$

The Randomdec signature for a zero-mean, stationary, Gaussian random process is simply the product of the correlation function and the trigger level.

$$D_{X_o}(\tau) = \rho_X(\tau) X_o = \frac{R_X(\tau)}{R_X(0)} X_o \tag{30}$$

For this special case the Randomdec signature is proportional to the autocorrelation function.

If $X(t)$ is the output of a linear time invariant system excited by a zero-mean, stationary, Gaussian random process, then $X(t)$ is also a zero-mean, stationary Gaussian random process. For such systems it is commonly believed that the Randomdec signature represents the transient decay of that system, to the initial conditions specified by

$$X(0) = X_0$$

$$\frac{dX(0)}{dt} = 0 \tag{31}$$

Since it has been shown that the Randomdec signature must be proportional to the autocorrelation function, then it can only be the transient decay of the system from the specified set of initial conditions under the restriction that the product of the correlation coefficient and the trigger level also represent the transient decay. This is only exactly true when the input to the system is white noise. However, for sharply tuned systems, such as a lightly damped single degree of freedom oscillator, a band limited spectrum often yields results that are to sufficient accuracy equivalent to the response to white noise. This applies as well to bandpass filtered random processes. However, the filter's characteristics are then part of the linear system being evaluated.

The Variance of the Randomdec Signature

In the previous sections the theoretical formulation of the Randomdec signature has been presented. As a practical matter, such a signature must be estimated from a limited number of finite length observations of an actual random process. The practical limitation on record length and number of records will introduce variance into the estimated signature. In this section an estimate of the variance of the Randomdec signature is obtained for the case that the time history X(t) is a zero mean, Gaussian, and stationary random process.

For a stationary random process, the theoretical Randomdec signature is a function of the delay τ and the trigger level $X(0) = X_o$, as stated in Equation (32).

$$D_{X_o}(\tau) = E[X(\tau) \,|\, X(0) = X_o] \tag{32}$$

For a finite number of samples $X_n(\tau)$ of the random process, an estimate of $D_{X_o}(\tau)$ may be obtained by

$$\hat{D}_{X_o}(\tau) = \frac{1}{N} \sum_{n=1}^{N} \left(X_n(\tau) \,|\, X_n(0) = X_o\right) \tag{33}$$

If each time history is sampled at m+1 discrete delay intervals, then the delay τ may be replaced by m, the number of discrete lags. This yields a discrete formulation for the estimate of $D_{X_o}(\tau)$.

$$\hat{D}_{X_o}(m) = \frac{1}{N} \sum_{n=1}^{N} \left(X_n(m) \,|\, X_n(0) = -X_o\right) \tag{34}$$

The expected value of the estimate may be found as follows.

$$E[\hat{D}_{X_o}(m)] = \frac{1}{N} E[\sum_{n=1}^{N} \left(X_n(m) \,|\, X_n(0) = X_o\right)] \tag{35}$$

$$= \frac{1}{N} \sum_{n=1}^{N} E[X_n(m) \,|\, X_n(0) = X_o] \tag{36}$$

$$E[\hat{D}_{X_o}(m)] = \frac{1}{N} \sum_{n=1}^{N} \int X_{n,m}\, p(X_{n,m}|X_o)\, dX_{n,m} \tag{37}$$

$$= \frac{R_X(m)}{R_X(0)}\, X_o \tag{38}$$

The last two equations follow directly from the analysis given in the previous section.

Therefore, it is concluded that

$$E[\hat{D}_{X_o}(m)] = \frac{R_X(m)}{R_X(0)}\, X_o \tag{39}$$

Since this is the discrete equivalent to the continuous formulation of Equation (30), the estimate of the Randomdec signature is unbiased.

To obtain the variance of the estimate requires first the estimate of the mean square of the Randomdec signature.

$$E[\hat{D}_{X_o}{}^2(m)] = \frac{1}{N^2}\, E[(\sum_{n=1}^{N} X_n(m)|X_n(0) = X_o)(\sum_{\ell=1}^{N} X_\ell(m)|X_\ell(0) = X_o)] \tag{40}$$

$$= \frac{1}{N^2} \sum_{n=1}^{N} \sum_{\ell=1}^{N} E[(X_n(m)|X_n(0) = X_o)(X_\ell(m)|X_\ell(0) = X_o)] \tag{41}$$

The expected value inside of the summation may also be expressed in probabilistic terms as follows.

$$E[X_n(m)|X_n(0) = X_o)(X_\ell(m)|X_\ell(0) = X_o)] =$$

$$= \iint X_{n,m}\, X_{\ell,m}\; p(X_{n,m}\,|X_o,\, X_{\ell,m}\,|X_o)\, dX_{n,m}\, dX_{\ell,m} \tag{42}$$

These results then imply that

$$E[\hat{D}_{X_o}^{\,2}(m)] = \frac{1}{N^2}\left\{ N\, R_X(0)\left(1 - \frac{R_X^{\,2}(m)}{R_X^{\,2}(o)}\right) + N\, \frac{R_X^{\,2}(m) X_o^{\,2}}{R_X^{\,2}(o)} + N(N-1)\, \frac{R_X^{\,2}(m)}{R_X^{\,2}(o)}\, X_o^{\,2} \right\} \quad (43)$$

The variance of the estimate is defined below in terms of the Randomdec signature.

$$\mathrm{Var}\,[\hat{D}_{X_o}^{\,2}(m)] = E[\hat{D}_{X_o}^{\,2}(m)] - E[\hat{D}_{X_o}(m)]^2 \quad (44)$$

Substitution of the previous results leads to

$$\mathrm{Var}[\hat{D}_{X_o}^{\,2}(m)] = \frac{1}{N^2}\left(N\, R_X(o)\left(1 - \frac{R_X^{\,2}(m)}{R_X^{\,2}(o)}\right) + N(N-1)\, \frac{R_X^{\,2}(m)}{R_X^{\,2}(o)}\, X_o^{\,2} \right)$$

$$- \frac{R_X^{\,2}(m)}{R_X^{\,2}(o)}\, X_o^{\,2} + \left(N\, \frac{R_X^{\,2}(m) X_o^{\,2}}{R_X^{\,2}(o)} \right) \frac{1}{N^2} \quad (45)$$

$$= \frac{1}{N}\, R_X(o) \left(1 - \frac{R_X^{\,2}(m)}{R_X^{\,2}(o)} \right) \quad (46)$$

$$= \frac{1}{N}\, R_X(o) \left(1 - \frac{D_{X_o}^{\,2}(m)}{X_o^{\,2}} \right) \quad (47)$$

For a zero mean, Gaussian, and stationary random process, the variance of the Randomdec signature decreases with N, the number of averages used in computing the estimate. As expected, for zero lag (m=0), the variance is zero. This is because the signature is forced to be equal to the trigger level. For very large lag, the variance increases to 1/N times the mean square of X(t), assuming $R_X(\tau) = 0$ as $\tau \to \infty$. It is important to note that the variance is independent of trigger level. This is true because the variance was calculated assuming that there was no noise in the measurement. If substantial noise were present, then the choice of too low a trigger level would result in false triggers which would grossly increase the variance of the estimate. The result of Equation (47) is valid for measurements with good signal to noise ratios and trigger levels substantially greater than the noise level.

This estimate of the variance was obtained with the assumption that individual Randomdec sample time histories were uncorrelated to one another. In practice, this is not generally the case. Sample time histories are typically acquired each time the random process crosses the specified trigger level. For reasonable trigger levels (such as on the order of the rms level of the random process) data acquisition will be initiated many times within the time frame of the decay length of $R_X(\tau)$. Thus each Randomdec sample may overlap many others, and the assumption of uncorrelated samples will not be valid. For a finite number of samples, correlation between samples will in general increase the variance of the estimate. Therefore, there is probably not much gained by triggering a new sample before data acquisition of the most recent one has been terminated. Users of Randomdec indicate that hundreds of samples of the highly overlapping type are required for convergence. It is suggested that convergence could be obtained with many fewer samples if overlapping were not allowed. This would not likely reduce the total data length required, but would cut down on the number of necessary computations. Cole [3] conducted Monte Carlo simulations and reports values of the variance for uncorrelated samples from the output of a single degree of freedom oscillator excited by band limited white noise. His values agree with the predictions of Equation (46).

An Application to an Offshore Structure

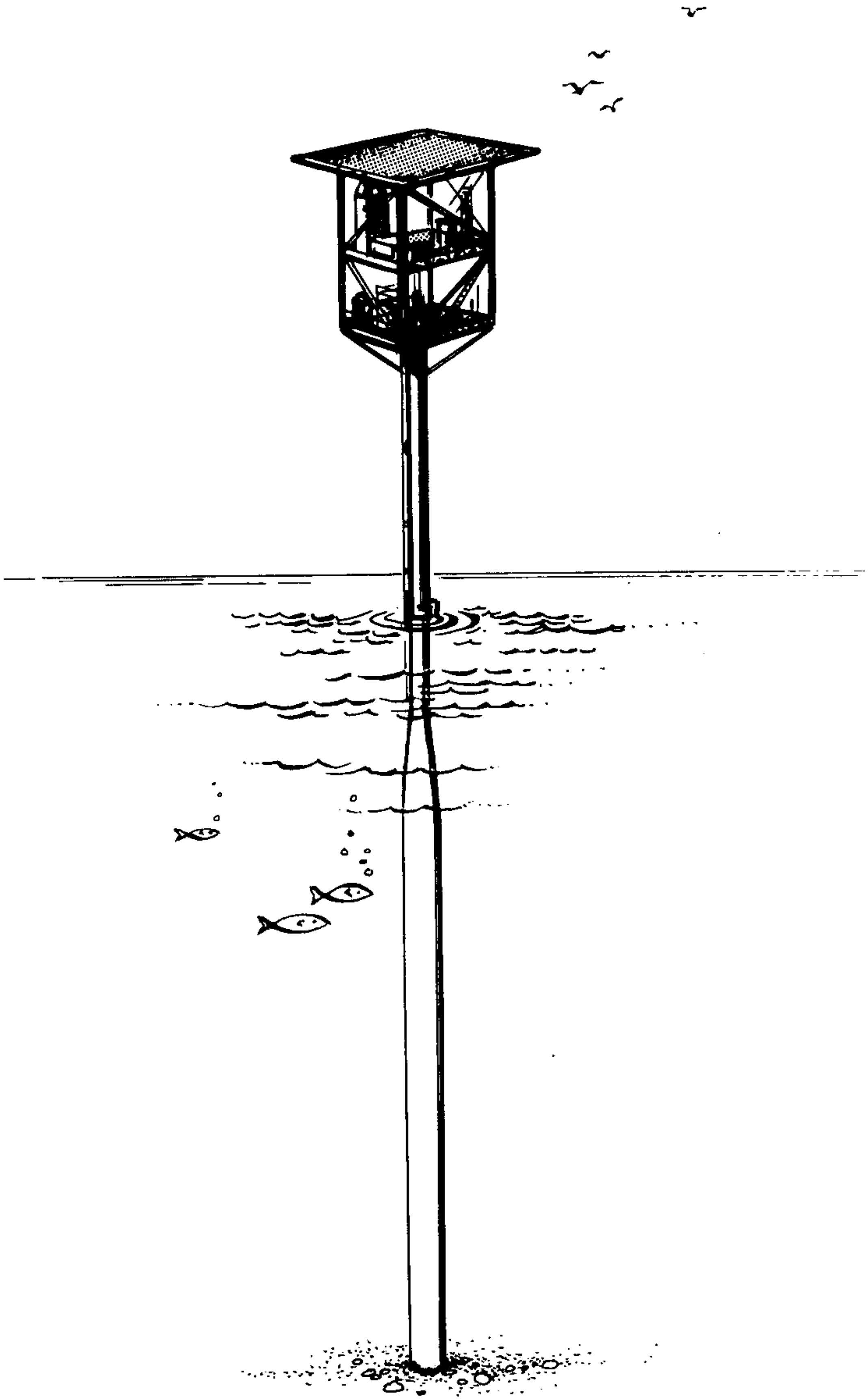
In March of 1980 authors Vandiver and Cook made acceleration response records on a single cylinder petroleum production platform depicted in Figure 1. This structure stands in 90 feet of water, is 4 feet in diameter at the waterline and extends to 76 feet above the water at the helicopter deck. It is very active dynamically with a lowest natural period of 3.28 seconds. Horizontal accelerations were recorded at several locations on the structure.

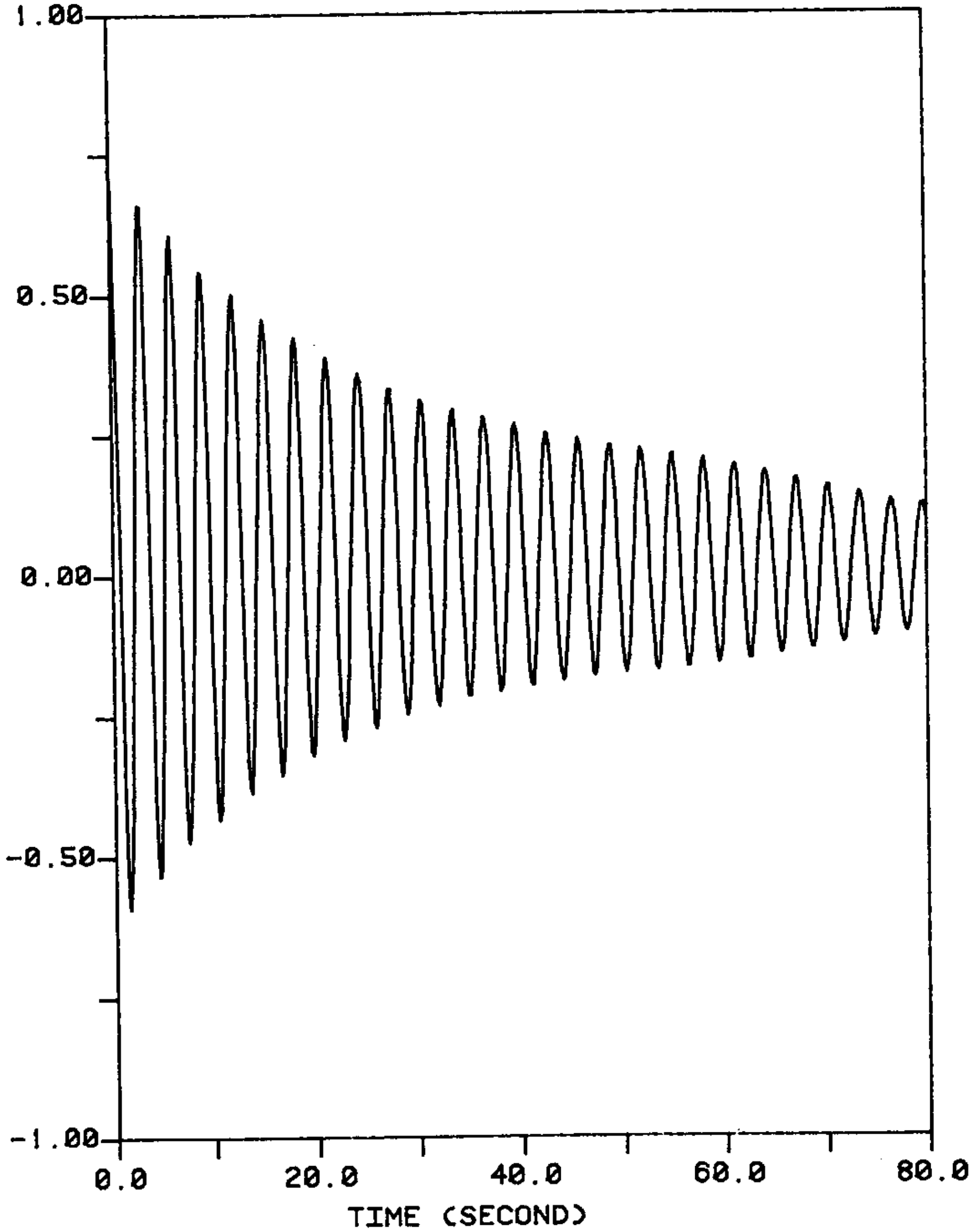
Randomdec and other more conventional analysis procedures have been applied to this data. Some of these results are presented here.

Figure 2 is an autocorrelation function computed from an acceleration time history of a location near the top of the structure. The maximum lag is 80 seconds and the total record length was 80 minutes, yielding a ratio of total record length to maximum lag of 60, a measure of the variance. The magnitude of this autocorrelation function has been normalized to force it to the same scale as the Randomdec signatures to be presented later. The recording was low pass filtered at 15 Hz to remove generator noise. No other filtering was employed. The data was very noise free and was clearly dominated by the response of the lowest bending natural mode of the structure. The autocorrelation function looks exactly as one would expect from a single degree of freedom oscillator excited by white noise.

A simple waveform averaging program was written for use on a GenRad Time Series Analysis system which is based on a Digital Equipment Corporation PDP 11/34 minicomputer.

The randomdec signatures were obtained by the following sampling procedure. The programmer specified the trigger level for the sample and



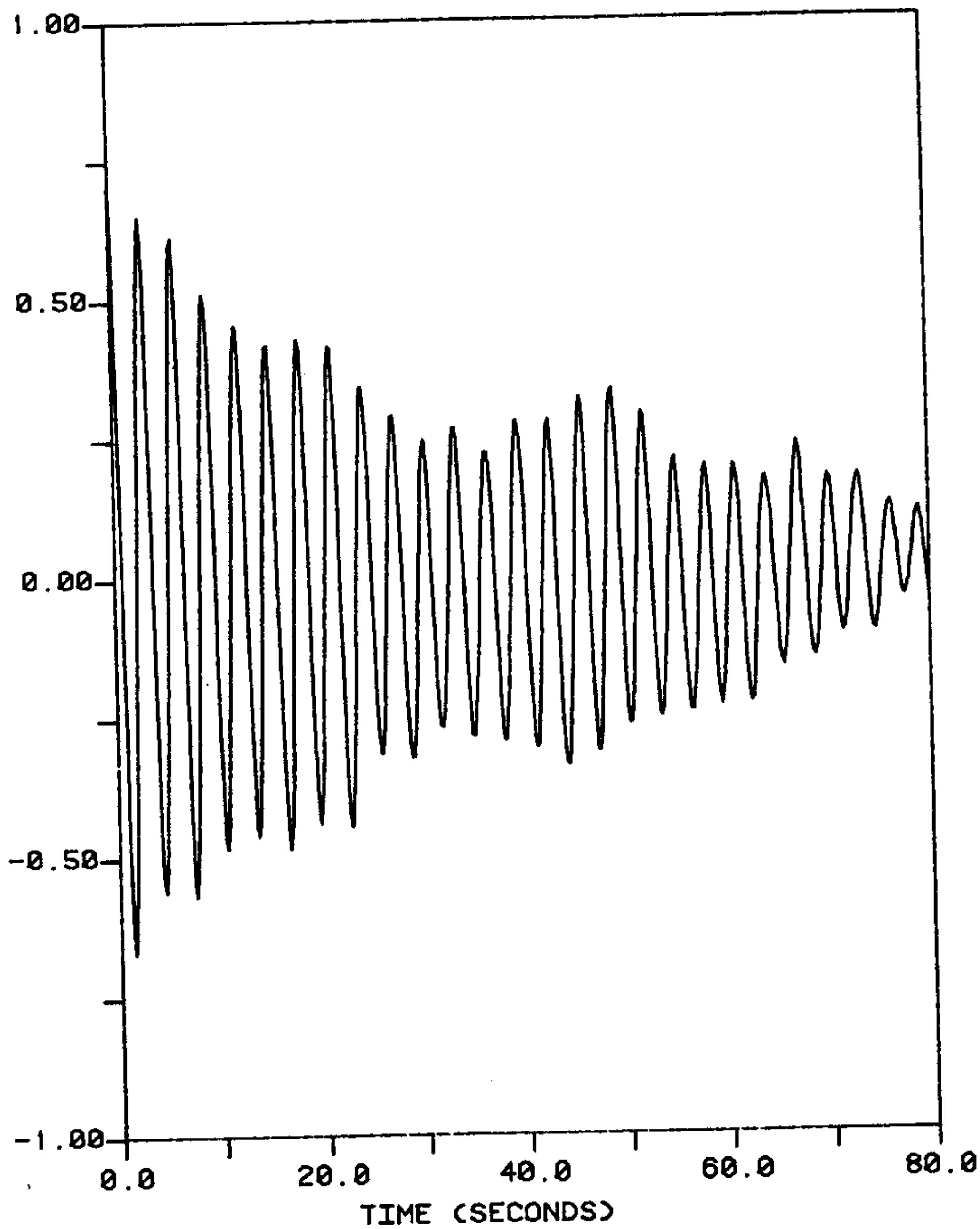

1.00
0.50
0.00
-0.50
-1.00
0.0
20.0
40.0
60.0
80.0
TIME (SECOND)

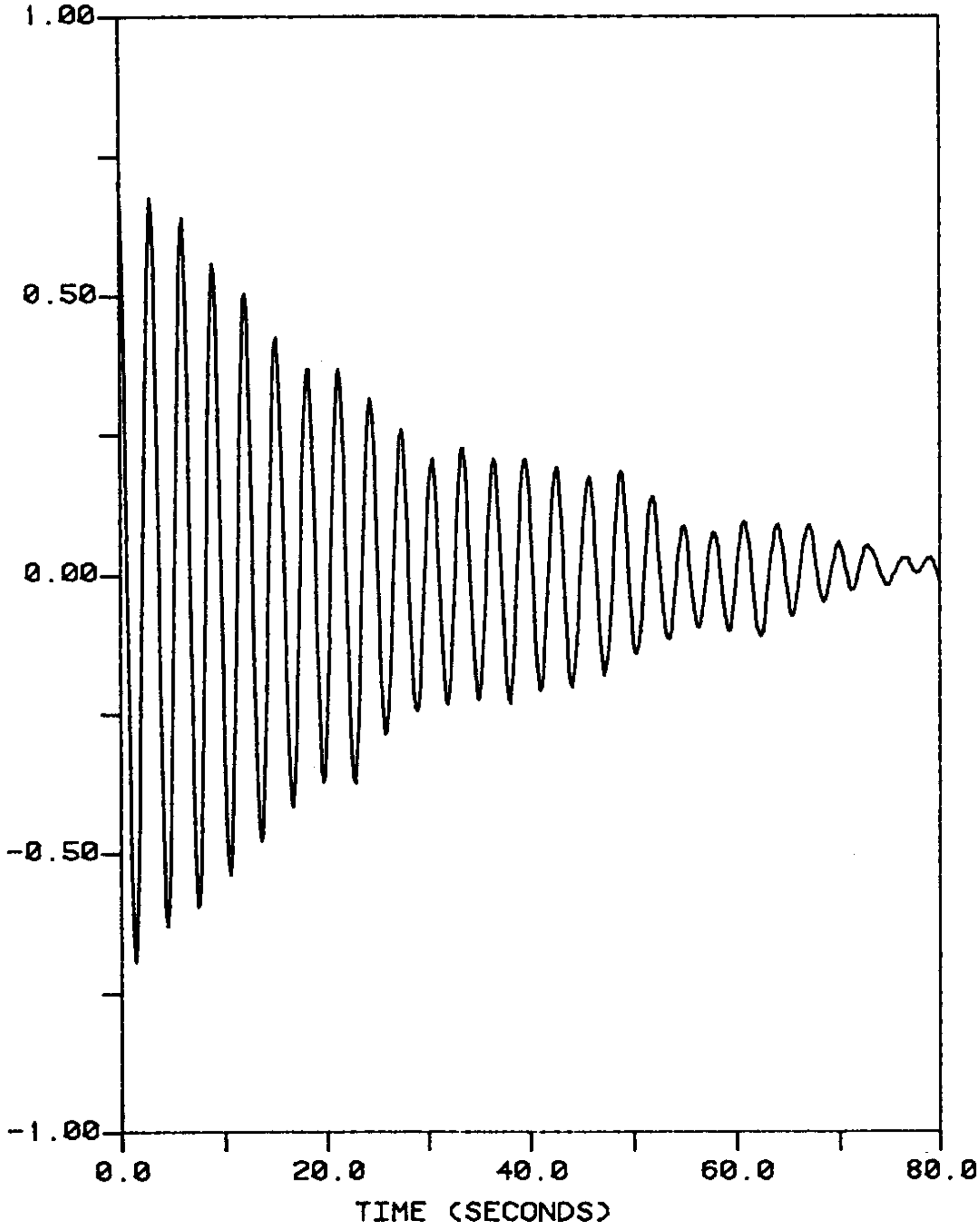
selected positive or negative slope. After each trigger a sample 80 seconds long was acquired. The sample was stored and when the trigger conditions were next satisfied, a new sample was begun. In this way two sequential samples could be considered to be essentially statistically independent, as can be tested by checking the value of the autocorrelation function at lags of 80 seconds and greater.

With this triggering procedure a maximum of 60 samples could be obtained from the total record for any specified trigger level and either + or - slope. Two sampling runs were made on the data. The trigger level was the same for each run, but the slope was plus for one and minus for the other. In the first case 50 samples were obtained in the second 43 samples were the result. The trigger level was set at approximately the r.m.s. level of the record.

Figure 3 shows the result of the ensemble average of 40 samples, 20 each with positive and negative slope. The result is clearly a long way from convergence to the shape of the autocorrelation function. Figure 4 shows the result of 86 averages, 43 of positive and 43 of negative slope. The result is considerably improved over the previous one, which had only 40 averages, although still a crude approximation to the shape of the autocorrelation function. Without requiring additional data, the only way to improve this result would be to obtain many highly correlated samples, for example, by triggering a new sample every time the trigger level is crossed with either positive or negative slope. This is in fact advocated in the Randomdec literature. Such techniques would be difficult to implement in our present software, and were not attempted.

Comparisons of the computed modal damping ratios using the autocorrelation data in Figure 2 and the Randomdec signature from Figure 4 are of interest.


1.00
0.50
0.00
-0.50
-1.00
0.0
20.0
40.0
60.0
80.0
TIME (SECONDS)


1.00
0.50
0.00
-0.50
-1.00
0.0
20.0
40.0
60.0
80.0
TIME (SECONDS)

Logarithmic decrement calculations over 20 cycles from each figure yield the following estimates for the modal damping ratio.

$\xi_1 = .010$		Autocorrelation
$\xi_1 = .016$		Randomdec
$\xi_1 = .014 \pm .0027$		MEM

The MEM estimate refers to a technique described in Reference [11], which uses the maximum entropy method (MEM) of spectral analysis to obtain an estimate of the damping ratio. That method also provides an estimate of the 95% confidence bounds on the estimated damping as shown above. Of the three techniques the authors place the most confidence in the MEM results.

CONCLUSIONS

The relationship between the autocorrelation of a random process and the most popular form of the Randomdec signature has been established. For a Gaussian random process, the Randomdec signature reduces to the product of the correlation function and the trigger level. For this case the variance of the estimated Randomdec signature is also found.

Because of the numerical simplicity of the Randomdec method, it provides a potentially useful way of obtaining the correlation function. In doing so, the shape of the autocorrelation function is obtained, at the sacrifice of the knowledge of the mean square value of the process. This is adequate for many purposes.

Acknowledgements

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References

[1] Cole, Henry A., Jr., "Failure Detection of a Space Shuttle Wing Flutter Model by Random Decrement," NASA TMX-62, 041, May 1971.

[2] Cole, Henry A., Jr., "Method and Apparatus for Measuring the Damping Characteristics of a Structure," United States Patent No. 3,620,069, Nov. 16, 1971.

[3] Cole, Henry A., Jr., "On-Line Failure Detection and Damping Measurement of Aerospace Structures by Random Decrement Signatures," NASA CR-2205, March 1973.

[4] Cole, Henry A., Jr., "On-the-line Analysis of Random Vibrations," AIAA Paper No. 68-288, AIAA/ASME Ninth Structures, Structural Dynamics and Materials Conference, Palm Springs, CA, 1968.

[5] Chang, C.S., "Study of Dynamic Characteristics of Aero-Elastic Systems Utilizing Randomdec Signatures," NASA CR-132563, 1975.

[6] Ibrahim, S.R., "Random Decrement Technique for Modal Identification of Structures," The AIAA Journal of Spacecraft and Rockets, Vol. 14, No. 11, 1977.

[7] Caldwell, D.W., "The Measurement of Damping and the Detection of Damage in Linear and Nonlinear Systems by the Random Decrement Technique," Ph.D. Thesis, University of Maryland Dept. of Mechanical Engineering, 1978.

[8] Ibrahim, S.R. and Mikulcik, E.C., "A Method for the Direct Identification of Vibration Parameters from Free Responses," Shock and Vibration Bulletin, Bulletin 47, 1977.

[9] Caughey, T.K., and Stumpf, H.J., "Transient Response of a Dynamic System Under Random Excitation," Journal of Applied Mechanics, December 1961, pp. 563-566.

[10] Crandall, S.H., and Mark, W.D., "Random Vibration in Mechanical Systems," Academic Press, New York and London, 1963.

[11] Campbell, R.B., and Vandiver, J.K., "The Determination of Modal Damping Ratios from Maximum Entropy Spectral Estimates", ASME paper No. 80-WA/DSC-29, presented at the ASME Winter Annual Meeting, Chicago, November 1980.

NOMENCLATURE

Single Degree of Freedom Parameters:

X, $\dot{X}$, $\ddot{X}$	Displacement, velocity and acceleration
M, K, R	Mass, spring and damper constants
F(t)	Force excitation
ξ	Damping ratio
ω_o	Undamped natural frequency
ω_1	Damped natural frequency
a,b	Initial displacement and velocity
$h(t-\tau)$	Impulse response function

General Symbols

X(t)	Time history of X
X_o	Initial value of X (t=0)
X_1, X_2	Random variables corresponding to possible values of X(t) at two different times
E[ ]	Expected value operator
$R_X(\tau)$	Autocorrelation function for X(t) at arbitrary lag τ
$R_X(0)$	Autocorrelation at $\tau = 0$
$\rho_X(\tau)$	Correlation function
$R_{XF}(t_1,t_2)$	Cross correlation between $X(t_1)$ and $F(t_2)$
$D_{X_o}(\tau)$	Randomdec signature
$\hat{D}_{X_o}(\tau)$	Estimated Randomdec signature
$p(X_1)$	Probability density function (pdf) for the random variable X_1

$p(X_1, F_2)$	Joint pdf for X_1, F_2
$p(F_2 \mid X_1)$	Conditioned pdf for F_2 given X_1
m_1, m_2	Mean values of two different Gaussian random variables
σ_1^2, σ_2^2	Mean squares of two Gaussian random variables
$X_n(\tau)$	nth sample time history of $X(t)$
N	total number of sample time histories.